

Worksheet for 2020-09-16

Problem 1. Let $f(x, y)$ and $g(u, v)$ be two functions, related by

$$g(u, v) = f(e^u + \sin v, e^u + \cos v).$$

Use the following values to calculate $g_u(0, 0)$ and $g_v(0, 0)$ (not all of the below values may be relevant!).

$$f(0, 0) = 3$$

$$g(0, 0) = 6$$

$$f_x(0, 0) = 4$$

$$f_y(0, 0) = 8$$

$$f(1, 2) = 6$$

$$g(1, 2) = 3$$

$$f_x(1, 2) = 2$$

$$f_y(1, 2) = 5$$

Problem 2. Consider the equation

$$x^7 - ax^6 + bx - 2 = 0.$$

If $(a, b) = (1, 2)$, then we have

$$x^7 - x^6 + 2x - 2 = 0$$

and you can check that $x = 1$ solves this equation. Now let's instead consider the equation

$$x^7 - 1.03x^6 + 2.06x - 2 = 0,$$

i.e. $(a, b) = (1.03, 2.06)$. Can you linearly approximate a solution for x to this equation? (Hint: use implicit differentiation to compute $\partial x/\partial a$ and $\partial x/\partial b$.)